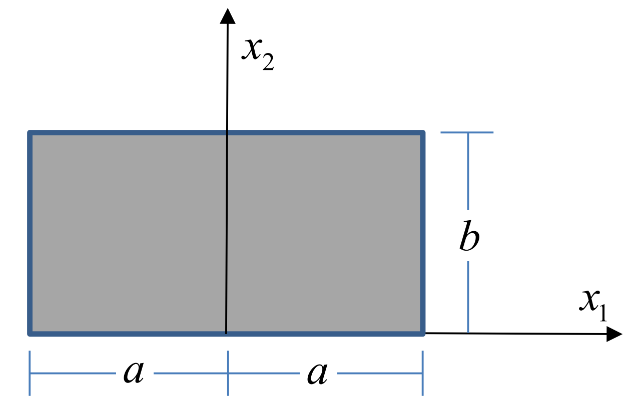
Exercise 1: For a solid in equilibrium with zero body forces, verify if the following stress field is possible.

$$\sigma_{11} = -2C_1x_1x_2;\quad \sigma_{22} = C_2x_3^2;\quad \sigma_{33} = C_2x_3^2$$
$$\sigma_{23} = 0;\quad \sigma_{12} = C_1(C_2 - x_2^2) + C_3x_1x_3;\quad \sigma_{13} = -C_3x_2$$

Exercise 2: For a thin plate (thickness *t*) without body forces, a stress field is given by,



$$\sigma_{11} = px_2x_1^3 - 2c_1x_2x_1 + c_2x_2$$
$$\sigma_{22} = px_1x_2^3 - 2px_1^3x_2$$
$$\sigma_{12} = -\frac{3}{2}px_1^2x_2^2 + c_1x_2^2 + \frac{1}{2}px_1^4 + c_3$$
$p, c_1, c_2,$ and c_3 are constants.

(a) Examine if it represents a solution for a thin plate of thickness *t* shown in the figure.
(b) Find the resultant normal and shear forces on the boundaries $x_2 = 0,\ x_2 = b$ of the plate.

Exercise 3: A bar with a constant cross-sectional area *A* and modulus *E,* is subjected to a uniform tensile stress as shown in the Figure below.

1. Find the strain energy *U* of the bar in terms of *P*, *l*, *A*, and *E*
2. Find the total elongation of the bar. What is the work done by the force *P*?
3. Find the total potential energy, defined as the difference of the strain energy and the work of the applied force, in terms of ℓ, *A, E* and strain $\varepsilon = \sigma / A$.

Exercise 4: A bar of uniform thickness (see Figure below), area *A* and modulus *E* is loaded with two forces, P_1 , P_2 .

(1) Apply the forces slowly and simultaneously until their final values. Show that the strain energy in the bar is,

$$U(P_1, P_2) = \frac{1}{2}\frac{P_1^2 L}{AE} + \frac{1}{2}\frac{P_2^2 L}{AE} + \frac{P_1 P_2 L}{AE}$$

(2) Apply the forces in the following manner: Firstly, apply the force P_1 from zero to its final values and then the force P_2 from zero to its final value. Calculate the energy of the bar at the end of the experiment. Repeat the same experiment with P_2 applied first and P_1 second. Show that the energy is the same and equal to that calculated in question (1)

(3) Show that the energy calculated in (1) or (2) is equal to the work done by the two applied forces, P_1, P_2

$$U(P_1, P_2) = \frac{1}{2}P_1 \Delta L_1 + \frac{1}{2}P_2 \Delta L_2,$$

where ΔL_1, ΔL_2 are the total elongations induced by P_1, P_2.

Problem 5: The strain energy of an elastic system, subjected to $P_1, P_2, ...P_n$ in equilibrium is given by the following expression (theorem of Clapeyron),

$$U = \frac{1}{2}a_{ij}P_iP_j \quad (i,j = 1,2,...n), \tag{a}$$

where the constants $a_{ij} = a_{ji}$ are the influence coefficients that relate the deflection at point *k* along the force P_k due to the entire system of applied forces *n*,

$$\delta_k = a_{kq}P_q. \tag{b}$$

Demonstrate that the derivative of (a) with respect to a force, gives the displacement the force along P_q,

$$\delta_k = \frac{\partial U}{\partial P_k} \tag{c}$$

Problem 6: Show that for a solid of volume ω in equilibrium, the following relation holds,

$$\int_{\omega} \sigma_{ij} u_{i,j} dv = \int_{\omega} \sigma_{ij} \varepsilon_{ij} dv$$

where $\sigma_{ij}, \varepsilon_{ij}$ are the stress and strain components in the solid.